Can cosmological observations uniquely determine the nature of dark energy ?

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(Dated: September 22, 2008)

The observational effect of all minimally coupled scalar field models of dark energy can be determined by the behavior of the following two parameters : (1) equation of state parameter w, which relates dark energy pressure to its energy density, and (2) effective speed of sound c_e^2, which relates dark energy pressure fluctuation to its density fluctuation. In this paper we show that these two parameters do not uniquely determine the form of a scalar field dark energy Lagrangian even after taking into account the perturbation in the scalar field. We present this result by showing that two different forms of scalar field Lagrangian can lead to the same values for these paired parameters. It is well known that from the background evolution the Lagrangian of the scalar field dark energy cannot be uniquely determined. The two models of dark energy presented in this paper are indistinguishable from the evolution of background as well as from the evolution of perturbations from a FRW metric.

PACS numbers: 95.36.+x

I. INTRODUCTION

The known form of matter such as radiation, atoms etc. can only make up 4% of the total matter content of the universe at the present epoch. The nature of the remaining 96%, of which about 23% is dark matter and 73% is some form of exotic dark energy driving the accelerated expansion of the universe, is still not completely understood[2]. One often invokes scalar fields to fill up the gap of this unknown form of matter especially for dark energy. Scalar field models of dark energy include quintessence[3], tachyon[5, 8], phantom[9, 10], k-essence[11] etc. For a detailed review on dark energy see Ref[1].

The present accelerated expansion of the universe could in fact be an indication of a nonzero value of the cosmological constant[12]. However, the present cosmological observations neither rule out a cosmological constant nor a scalar field as a candidate for dark energy[13]. Cosmological constant can be ruled out if a definitive detection of perturbations in dark energy is made[16].

Dark energy influences cosmological observations such as luminosity distance and angular diameter distance through its effect on the rate of expansion of the universe. For scalar field models of dark energy, the perturbations in scalar fields affect the evolutions of the metric perturbations from the FRW metric, which will consequently show up in the ISW effect[17]. All of these effects of scalar field dark energy on cosmic expansion rate as well as on the ISW effect can be characterized by two parameters : (1) equation of state parameter given by $w = p/\rho$ and (2) the effective speed of sound c_e^2 which relates pressure fluctuations to density fluctuations. This implies that from cosmological observations one can in principle esti-

mate w[13, 14, 15] and c_e^2[18]. Evolution of the equation of state parameter $w(t)$ alone cannot uniquely determine the Lagrangian of the scalar field dark energy[6]. This implies that from the background evolution $a(t)$ one cannot determine the form of the scalar field Lagrangian uniquely [4, 5, 6, 7].

The aim of this paper is to investigate whether the form of the scalar field dark energy Lagrangian is uniquely determined if we know the background evolution $a(t)$ as well as the evolution of the metric perturbation. In other words the question we are addressing in this paper is whether the values of w and c_e^2 uniquely fix the form of the scalar field dark energy Lagrangian? We show that the answer to this is no. We demonstrate this by showing that two different forms of scalar field Lagrangian given by (1) $\mathcal{L}_1 = X^\alpha - V_1(\phi)$ and (2) $\mathcal{L}_2 = -V_2(\phi)(1 - 2X)^\beta$, where α and β are constants and $X = (1/2)\partial_\mu\phi\partial^\mu\phi$, can lead to the same values of w and c_e^2. This implies that the evolution of the background as well as the metric perturbation is identical in both of these models. This is achieved by appropriately choosing the value of β for a given value of α. We illustrate with this example that if the present accelerated expansion of the universe is not due to a cosmological constant or quintessence then it will be impossible to uniquely determine the nature of dark energy from cosmological observations.

In this paper we work in the longitudinal gauge. This paper is organized in the following way : In Sec. II we show that the evolution of the scale factor $a(t)$ and metric perturbation $\Phi(\vec{x}, t)$ in the longitudinal gauge is determined by equation of state parameter w and the effective speed of sound c_e^2 of dark energy. In Sec. III we present a model of generalized quintessence dark energy. In section IV we present a model of generalized tachyon dark energy which influences cosmological observations in exactly the same way as that by the model presented in Sec. III. Sec. V summarizes the results. In addition, we present in the appendix a generalized closed set of cosmo-

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logical perturbation equations applicable to perfect fluid and scalar fields. In this paper we work in natural units defined as $\hbar = c = 1$.

II. DARK ENERGY PARAMETERS w AND c_e^2

We shall consider a universe with minimally coupled pressureless matter and scalar field dark energy with Lagrangian $\mathcal{L} = \mathcal{L}(X, \phi)$ which is a general function of the kinetic term $X = (1/2)\partial_\mu\phi\partial^\mu\phi$ and the field ϕ. For this system, scalar metric perturbation in the longitudinal gauge is given by [19, 20, 21]:

$$ds^2 = (1 + 2\Phi) dt^2 - a^2(t) (1 - 2\Phi) \left[dx^2 + dy^2 + dz^2\right] \tag{1}$$

We assume a $k = 0$ (flat) universe. The evolution of the scale factor $a(t)$ is determined by the following Friedmann equation :

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \left[\rho_{m0}a^{-3} + \bar{\rho}_{de}(a)\right] \tag{2}$$

where ρ_{m0} is the density of the pressureless matter at the present epoch and $\bar{\rho}_{de}(a)$ is the background dark energy density[1] given by :

$$\bar{\rho}_{de}(a) = \rho_{de0} \exp\left[-3\int(1+w)\frac{da}{a}\right] \tag{3}$$

where ρ_{de0} is the homogeneous component of the dark energy density at the present epoch and w is the equation of state parameter of the dark energy which can be determined by the Lagrangian of the scalar field. This is given by :

$$w = \frac{\bar{\mathcal{L}}(\bar{X}, \bar{\phi})}{2\frac{\partial\bar{\mathcal{L}}}{\partial\bar{X}}\bar{X} - \bar{\mathcal{L}}(\bar{X}, \bar{\phi})} \tag{4}$$

where $\bar{X} = (1/2)\dot{\bar{\phi}}^2$ and $\bar{\mathcal{L}} = \bar{\mathcal{L}}(\bar{X}, \bar{\phi})$ is the Lagrangian of the background field[2] $\bar{\phi}$ which is only a function of time. For example we have for a canonical scalar field $\bar{\mathcal{L}}(\bar{X}, \bar{\phi}) = (1/2)\dot{\bar{\phi}}(t)^2 - V(\bar{\phi})$. The evolution of the scalar field whose dynamics is described by the Lagrangian $\bar{\mathcal{L}}(\bar{X}, \bar{\phi})$ is determined by the field equation :

$$\left[\frac{\partial\bar{\mathcal{L}}}{\partial\bar{X}}\dot{\bar{\phi}}\right]^{\cdot} + 3H\left(\frac{\partial\bar{\mathcal{L}}}{\partial\bar{X}}\right)\dot{\bar{\phi}} + \frac{\partial\bar{\mathcal{L}}}{\partial\bar{\phi}} = 0 \tag{5}$$

Fractional perturbation in the matter density and in

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[1] All the variable denoted with an over bar such as $\bar{\rho}$ and $\bar{p}$ corresponds to their average value on the background space time $ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right]$

[2] Perturbation in the scalar field is defined as $\phi(\vec{x}, t) = \bar{\phi}(t) + \delta\phi(\vec{x}, t)$

dark energy are defined as :

$$\delta_m \equiv \frac{\delta\rho_m}{\bar{\rho}_m} \tag{6}$$

$$\delta_{de} \equiv \frac{\delta\rho_{de}}{\bar{\rho}_{de}} \tag{7}$$

The evolution of metric perturbation Φ, matter perturbation δ_m and perturbation in dark energy δ_{de} is determined by the following set of equations :

$$3\frac{\dot{a}^2}{a^2}\Phi + 3\frac{\dot{a}}{a}\dot{\Phi} + \frac{k^2\Phi}{a^2} = -4\pi G\left[\rho_{mo}a^{-3}\delta_m + \bar{\rho}_{de}(a)\delta_{de}\right] \tag{8}$$

$$\dot{\delta}_m = k^2 u_m + 3\dot{\Phi} \tag{9}$$

$$\dot{u}_m = -2Hu_m - \frac{\Phi}{a^2} \tag{10}$$

$$\dot{\delta}_{de} = (1+w)k^2 u_{de} + 3H\left(w - c_e^2\right)\delta_{de} + 9H^2$$
$$\times (1+w)\left[c_e^2 - c_a^2\right]a^2 u_{de} + 3(1+w)\dot{\Phi} \tag{11}$$

$$\dot{u}_{de} = -H\left(2 - 3c_e^2\right)u_{de} - \frac{c_e^2\delta_{de}}{a^2(1+w)} - \frac{\Phi}{a^2} \tag{12}$$

where $H \equiv \dot{a}/a$,

$$c_a^2 = \frac{\dot{\bar{p}}_{de}}{\dot{\bar{\rho}}_{de}} = w - \frac{\dot{w}}{3H(1+w)\bar{\rho}_{de}} \tag{13}$$

is the adiabatic sound speed and c_e^2 is the effective sound speed given by [22] :

$$c_e^2 = \frac{\frac{\partial\bar{\mathcal{L}}}{\partial\bar{X}}}{\frac{\partial\bar{\mathcal{L}}}{\partial\bar{X}} + 2\bar{X}\frac{\partial^2\bar{\mathcal{L}}}{\partial\bar{X}^2}} \tag{14}$$

The effective speed of sound c_e^2 relates dark energy pressure fluctuation to its density fluctuation in the following way [23, 24] :

$$\delta p_{de} = c_e^2\delta\rho_{de} - 3H(\bar{\rho}_{de} + \bar{p}_{de})a^2 u_{de}\left[c_e^2 - c_a^2\right] \tag{15}$$

Eq.(8) is the time-time component of the linearized Einstein equation $\delta G^\mu_\nu = \kappa\delta T^\mu_\nu$. Eqs.(10) to (12) follow from the covariant conservation equation $T^\mu_{\nu\,;\,\mu} = 0$, which are individually valid for both matter and dark energy since they are minimally coupled. In Eqs.(10) to (12), u_m and u_{de} are the potential for the respective peculiar velocity[3](or velocity perturbation) δu^i in the perturbed energy momentum tensor δT^μ_ν.

In the gauge in which $u_{de} = B = 0$, where B is the scalar metric perturbation corresponding to δg_{0i}, the effective speed of sound of dark energy is the ratio of its pressure fluctuation to its density fluctuation. For scalar fields, in general, $c_e^2 \neq c_a^2$. However, for perfect fluids these two sound speeds coincides. The fact that c_e^2 does

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[3] u_m and u_{de} are defined in Appendix

not (in general) coincide with c_a^2 is the consequence of non zero intrinsic entropy perturbation of the scalar field.

For a given value of the equation of state parameter w (which could in general be a function of epoch), Eq.(2) and Eq.(3) can be solved to determine the evolution of the scale factor $a(t)$. The perturbation equations [Eqs.(8) to (12)] are affected by both w and c_e^2. This implies that w and c_e^2 are the two parameters of the scalar field dark energy which determines the solution $a(t)$ and $\Phi(\vec{x}, t)$ in the line element (1). The question we are addressing in this paper is whether two different forms of the Lagrangian $\mathcal{L}(X, \phi)$ lead to the same set of w and c_e^2. We will argue that this is indeed true by showing that two different forms of the Lagrangian namely generalized quintessence with $\mathcal{L}_1 = X^\alpha - V_1(\phi)$ and generalized tachyon with $\mathcal{L}_2 = -V_2(\phi)\,(1 - 2X)^\beta$ lead to the same w and c_e^2. In this paper we only consider the case when both w and c_e^2 are constant.

III. GENERALIZED QUINTESSENCE DARK ENERGY

We will first consider a generalized quintessence model of dark energy with Lagrangian given by [25]:

$$\mathcal{L}_1 = X^\alpha - V_1(\phi) \tag{16}$$

where α is a constant. If $\alpha = 1$, then this Lagrangian corresponds to the canonical scalar field or quintessence dark energy [3].

We shall reconstruct the form of the potential $V_1(\phi)$ such that it leads to the solution for which $w = $ constant. This would then imply that $\bar{\rho}_{de}(a) = \bar{\rho}_{de0} a^{-3(1+w)}$, where $\bar{\rho}_{de0}$ is the dark energy density at the present epoch. The Friedmann equation [Eq.(2)] would then becomes :

$$H^2 = H_0^2 \left[\Omega_{mo} a^{-3} + \Omega_{de0} a^{-3(1+w)} \right] \tag{17}$$

where H_0 is the Hubble parameter at the present epoch, $\Omega_{mo} = 8\pi G \bar{\rho}_{m0}/(3H_0^2)$ and $\Omega_{deo} = 8\pi G \bar{\rho}_{de0}/(3H_0^2)$ are the dimensionless density parameters at the present epoch of matter and dark energy respectively.

From the Lagrangian (16), it follows that :

$$\bar{\rho}_{de} = \left(\frac{2\alpha - 1}{2\alpha} \right) \dot{\bar{\phi}}^{2\alpha} + V_1(\bar{\phi}) \tag{18}$$

$$\bar{p}_{de} = \frac{\dot{\bar{\phi}}^{2\alpha}}{2\alpha} - V_1(\bar{\phi}) \tag{19}$$

From Eqs.(17), (18) and (19) we obtain :

$$\begin{aligned} \frac{d\bar{\phi}}{da} &= \sqrt{2} \left[\frac{3H_0^{2(1-\alpha)} M_p^2 \,(1 + w)\, \Omega_{deo}}{2\alpha} \right]^{\frac{1}{2\alpha}} \\ &\times \frac{a^{-\lambda}}{\sqrt{\Omega_{mo} a^{-3} + \Omega_{de0} a^{-3(1+w)}}} \end{aligned} \tag{20}$$
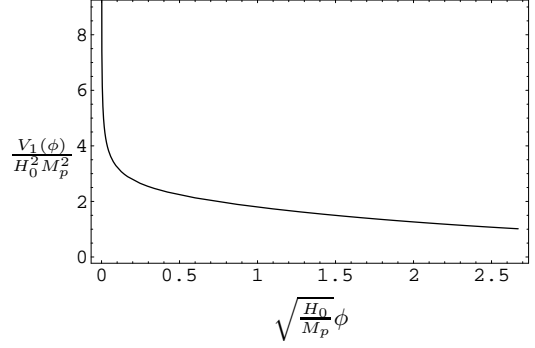

FIG. 1: The behavior of the generalized quintessence potential $V_1(\phi)$ with ϕ for $w = -0.9$ and for $\alpha = 2$. For this model $c_e^2 = 1/3$.

$$V_1(a) = 3 \left[\frac{1 - (2\alpha - 1)w}{2\alpha} \right] \frac{H_0^2 M_p^2 \Omega_{de0}}{a^{3(1+w)}} \tag{21}$$

where $M_p = 1/\sqrt{8\pi G}$ is the Planck mass and

$$\lambda = \frac{3\,(1 + w) + 2\alpha}{2\alpha} \tag{22}$$

In scalar field dominated universe with $\Omega_{de0} = 1$, Eq.(20) can be analytically solved to obtain the solution $a(\phi)$. Substituting this solution in Eq.(21) leads to the following form of the potential :

$$V_1(\phi) = \frac{V_0}{\phi^n} \tag{23}$$

where

$$n = \frac{2\alpha}{\alpha - 1} \tag{24}$$

This form of the potential [Eq.(23)] corresponds to the case when $\alpha \neq 1$. For $\alpha = 1$, which corresponds to standard scalar field or quintessence, Eqs.(20) and (21) lead to an exponential form of the potential in the scalar field dominated universe. In this paper we will only consider the case when the parameter $\alpha \neq 1$.

A realistic model of the universe consistent with observations would require $\Omega_{mo} = 0.27$, $\Omega_{deo} = 0.73$ and w close to minus one [32, 33, 34]. We numerically obtain the form of the potential from Eqs.(20) and (21) with these values for Ω_{mo} and Ω_{deo}. This is shown in Fig.1 and it corresponds to the same class of the potentials described in Eq.(23). This form of the potential leads to a solution for which the equation of state parameter $w = -0.9$ in a universe with pressureless matter and scalar field dark energy with Lagrangian of the form given by Eq.(16) and with the value of the parameter $\alpha = 2$. Eq.(20) has been numerically integrated with the initial condition $\sqrt{H_0/M_p}\phi = 10^{-4}$ at $a = 10^{-3}$.

Since the equation of state parameter w in this model is constant, the adiabatic sound speed $c_a^2 = w$ and the effective sound speed defined in Eq.(14) is given by [26]:

$$c_e^2 = \frac{1}{2\alpha - 1} \qquad (25)$$

Given any value of c_e^2, assuming it to be constant, we can appropriately choose α given by Eq.(25). For $\alpha > 1$, the effective speed of sound is less than the speed of light $i.e.$ $c_e^2 < 1$[30]. However, k-essence dark energy in general admit solutions for which the speed of sound is greater than the speed of light [28, 31]. The question of whether such superluminal propagation of perturbation on classical background is acceptable or not is debated in the literature (see for instance Refs.[27, 29, 30, 31]).

IV. GENERALIZED TACHYON DARK ENERGY

In this section we will consider a model of generalized tachyon dark energy with Lagrangian of the form :

$$\mathcal{L}_2 = -V_2(\phi)\left(1 - 2X\right)^\beta \qquad (26)$$

For $\beta = 1/2$, this form of the Lagrangian would become the usual DBI form of the Lagrangian [5, 8]. A model of generalized tachyon field with constant potential can be found in Refs.[35, 36]. In this paper, we will reconstruct a form of the potential $V_2(\phi)$ such that it leads to the solution for which the equation of state parameter is constant. The value of the parameter β would then be fixed such that the effective speed of sound c_e^2 for both forms of the Lagrangian given by Eq.(16) and Eq.(26) is exactly the same.

From the Lagrangian given by Eq.(26) we obtain :

$$\bar{\rho}_{de} = V_2(\bar{\phi})\left[(2\beta - 1)\dot{\bar{\phi}}^2 + 1\right]\left(1 - \dot{\bar{\phi}}^2\right)^{\beta-1} \quad (27)$$

$$\bar{p}_{de} = -V_2(\bar{\phi})\left(1 - \dot{\bar{\phi}}^2\right)^\beta \qquad (28)$$

For a constant value of the equation of state parameter w, Eqs.(27) and (28) leads to the following equations :

$$\begin{aligned}
\frac{d\bar{\phi}}{da} &= \frac{1}{H_o}\sqrt{\frac{1+w}{1 - w(2\beta - 1)}} \\
&\times \frac{1}{a\sqrt{\Omega_{mo}a^{-3} + \Omega_{de0}a^{-3(1+w)}}}
\end{aligned} \qquad (29)$$

$$V_2(a) = -3w\left[\frac{w(2\beta - 1) - 1}{2w\beta}\right]^\beta \frac{H_0^2 M_p^2 \Omega_{de0}}{a^{3(1+w)}} \qquad (30)$$

In a scalar field dominated universe with $\Omega_{de} = 1$, Eqs.(29) and (30) lead to the following form of the potential :

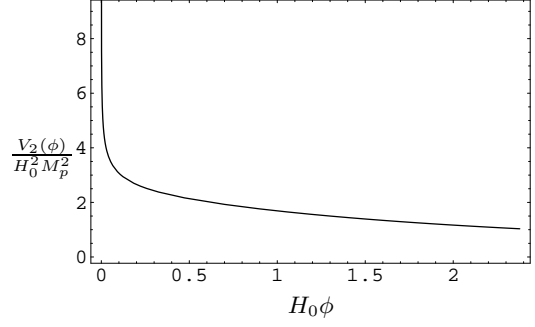$$V_2(\phi) = \frac{V_0}{\phi^2} \qquad (31)$$

FIG. 2: A form of the generalized tachyon potential $V_2(\phi)$ for $w = -0.9$ and for $\beta = 1/19$. For this model $c_e^2 = 1/3$.

This form of the potential with Lagrangian of the form given by Eq.(26) leads to a solution in the scalar field dominated universe for which the equation of state parameter is constant. However, for a realistic model of the universe to be consistent with the observation would require that $\Omega_{mo} = 0.27$, $\Omega_{deo} = 0.73$. In this case we numerically obtain the form of the potential from Eqs.(29) and (30). The form of the potential thus obtained is shown in Fig. 2. Eq.(20) has been numerically integrated with the initial condition $H_0\phi = 10^{-4}$ at $a = 10^{-3}$.

From the Lagrangian Eq.(26), the effective speed of sound defined in Eq.(14) would become :

$$c_e^2 = \frac{-\beta w}{(1 - \beta) + w(1 - 2\beta)} \qquad (32)$$

For a given value of the parameter α in the Lagrangian (16) and for a given value of the equation of state parameter w, if we choose the value of the parameter β such that:

$$\beta = \frac{1 + w}{1 + w(3 - 2\alpha)} \qquad (33)$$

then the effective speed of sound for both forms of the Lagrangian is exactly the same. For example if $\alpha = 2$ and $w = -0.9$ then we require $\beta = 1/19$. For $w = -0.8$ and $\alpha = 2$, the required value of the parameter is $\beta = 1/9$.

Since the equation of state parameter w for both models of dark energy presented in this paper is the same, its effect on the cosmic expansion rate $a(t)$ determined by Eqs.(2) and (3) would be identical. Also, since the effective speed of sound c_e^2 is the same for both models, its effect on the metric perturbation $\Phi(t, \vec{x})$ as well as on the matter power spectrum determined by Eqs.(8) to (12) would be identical. Hence, if we fix the value of the parameter β using Eq.(33), then for a given w and α, the two forms of the Lagrangian would be observationally indistinguishable.

If $c_e^2 = 1$, then it turns out from Eqs.(25) and (33) that $\alpha = \beta = 1$. Only in this case the two form of the Lagrangian given by Eqs.(16) and (26) would be related

through a redefinition of the fields. Also the degeneracy in the model space does not hold if the present accelerated expansion is driven by the cosmological constant for which $w = -1$.

Hence, from cosmological observations we might be able to determine the value of the dark energy parameters w and c_e^2. However, with these values of w and c_e^2 we will not be able to determine uniquely the form of the scalar field Lagrangian if $w \neq -1$ and $c_e^2 \neq 1$. Hence, we emphasize that besides confronting models of dark energy with cosmological observations, we must also devise some mechanism (or experiments) to determine its nature directly. For example, a model of dark energy described in Ref.[38] can likely be tested at the LHC.

V. CONCLUSIONS

It is demonstrated in Refs.[4, 5, 6, 7] that a given background evolution $a(t)$ can be obtained from fundamentally different forms of the scalar field Lagrangian. In this paper we have discussed two scalar field models which are indistinguishable not only from the background evolution $a(t)$ but also from the evolution of metric perturbation $\Phi(t, \vec{x})$ in the longitudinal gauge. We have demonstrated this by showing that two different models of scalar field dark energy can lead to the same set of two parameters w and c_e^2.

In this paper we have considered two different models of dark energy with a Lagrangian of the form $\mathcal{L}_1 = X^\alpha - V_1(\phi)$ and $\mathcal{L}_2 = -V_2(\phi)(1-2X)^\beta$. We have reconstructed the form of the two potentials $V_1(\phi)$ and $V_2(\phi)$ such that in both models $w = -0.9$. The two constants α and β were fixed such that in both models $c_e^2 = 1/3$. In fact, from these two forms of the Lagrangian it is possible to reconstruct a model of dark energy with any value of $w > -1$ and $c_e^2 < 1$, assuming that both w and c_e^2 are constant.

A universe with roughly about 27% dark matter with negligible pressure and 73% scalar field dark energy with either of the above two Lagrangians will lead to the same solution for the scale factor $a(t)$ and for the metric perturbations about FRW metric, for the same set of initial conditions. Hence, the observable effects of these two models of dark energy will be identical. With this example, we conclude that *it is impossible to uniquely determine the nature of dark energy from cosmological observations if $w \neq -1$ and $c_e^2 \neq 1$* (for constant w and c_e^2).

All of these results emphasize the fact that besides indirectly determining the nature of dark energy through its effect on the cosmic expansion rate, matter power spectrum, ISW effect, etc., we must also devise some mechanism (or experiments) to determine its nature directly.

In this paper we have only considered the case when both w and c_e^2 are constant. The generalization of this result for $w = w(t)$ and $c_e^2 = c_e^2(t)$, *i.e* when both w and c_e^2 are function of epoch, is in progress.

APPENDIX A: COSMOLOGICAL PERTURBATION EQUATIONS FOR PERFECT FLUID/ SCALAR FIELDS

In this appendix, we shall present a closed set of cosmological perturbation equations applicable to perfect fluids and scalar fields.

Scalar metric perturbations describing a perturbed spatially flat FRW line element is given by [19, 20, 21] :

$$
\begin{aligned}
ds^2 &= (1 + 2A) \, dt^2 - 2aB_{,i}dx^i dt \\
&\quad - a^2 \left[(1 - 2\psi)\delta_{ij} + 2E_{,ij} \right] dx^i dx^j
\end{aligned} \quad \text{(A1)}
$$

where A , ψ , B and E are 3−space scalars.

For most content of the universe such as perfect fluid and scalar fields the energy momentum tensor can be expressed as :

$$
T^\mu{}_\nu = (\rho + p) \, u^\mu u_\nu - p\delta^\mu{}_\nu \quad \text{(A2)}
$$

where ρ is the energy density, p is the pressure and u^μ is the four velocity field.

We define the perturbations in the energy density ρ, pressure p and the four velocity field u^μ in the following way :

$$
\begin{aligned}
\rho(t, \vec{x}) &= \bar{\rho}(t) + \delta\rho(t, \vec{x}) & \text{(A3)} \\
p(t, \vec{x}) &= \bar{p}(t) + \delta p(t, \vec{x}) & \text{(A4)} \\
u^\mu &= \bar{u}^\mu + \delta u^\mu & \text{(A5)}
\end{aligned}
$$

where $\bar{u}^\mu = [1, 0, 0, 0]$ and since $u^\mu u_\mu = 1$, it follows that $\delta u_0 = -\delta u^0 = A$.

The spatial part of the perturbations in the four velocity field δu^i is the peculiar velocity which can be written as gradient of a scalar :

$$
\delta u^i = \delta^{ij} u_{,j} \quad \text{(A6)}
$$

which implies that $\delta T^0{}_i = (\bar{\rho} + \bar{p}) \, u_{,i}$ and

$$
\delta T^i{}_0 = -a^2 (\bar{\rho} + \bar{p}) \left[u_{,i} + a^{-1}B_{,i} \right] \quad \text{(A7)}
$$

Under infinitesimal coordinate transformation defined as $x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu$, where $\xi^\mu = (\xi^0, \delta^{ij}\xi_{,j})$, the variables describing metric perturbations A, ψ, B and E transforms as :

$$
\begin{aligned}
\widetilde{A} &= A - \dot{\xi}^o & \text{(A8)} \\
\widetilde{\psi} &= \psi - H\xi^o & \text{(A9)} \\
\widetilde{B} &= B + a^{-1}\xi^o - a\dot{\xi} & \text{(A10)} \\
\widetilde{E} &= E - \xi & \text{(A11)}
\end{aligned}
$$

Similarly the variables describing matter perturbations $\delta\rho$, δp and u transforms as :

$$\widetilde{\delta\rho} = \delta\rho - \dot{\rho}_o\xi^o \qquad (A12)$$

$$\widetilde{\delta p} = \delta p - \dot{p}_o\xi^o \qquad (A13)$$

$$\widetilde{u} = u + \dot{\xi} \qquad (A14)$$

Linearized Einstein's Equation $\delta G^\mu{}_\nu = \kappa\delta T^\mu{}_\nu$, which relates variables describing metric perturbations (A, ψ, B and E) to the variables describing matter perturbations (ρ, δp and u) is given by :

$$3H^2A + H\dot{\psi} + \frac{k^2}{a^2}\left[\psi - H\left(aB - a^2\dot{E}\right)\right] = -4\pi G\delta\rho \qquad (A15)$$

$$\ddot{\psi} + 3H\dot{\psi} + H\dot{A} + \left(2\dot{H} + 3H^2\right)A = 4\pi G\delta p \qquad (A16)$$

$$\dot{\psi} + HA = -4\pi Ga^2\left(\bar{\rho} + \bar{p}\right)\left[u + a^{-1}B\right] \qquad (A17)$$

In Eq.(A16) we have used the fact that for perfect fluids and for scalar fields anisotropic stress is zero $i.e$ $\delta T^i{}_j \propto \delta^i_j$ for i, $j = 1$, 2, 3. This implies that :

$$A - \psi + H\left(aB - a^2\dot{E}\right) + \left(aB - a^2\dot{E}\right)^{\cdot} = 0 \qquad (A18)$$

The covariant conservation equation $T^\mu{}_{\nu\,;\,\mu} = 0$ leads to the following equations :

$$\dot{\delta\rho} = \left(\bar{\rho} + \bar{p}\right)k^2u - 3H\left(\delta\rho + \delta p\right) + \left(\bar{\rho} + \bar{p}\right)\left[3\dot{\psi} + k^2\dot{E}\right] \qquad (A19)$$

$$\dot{u} = -H\left(2 - 3c_a^2\right)\left[u + \frac{B}{a}\right] - \frac{\delta p}{a^2\left(\bar{\rho} + \bar{p}\right)} - \frac{(aB)^{\cdot}}{a^2} - \frac{A}{a^2} \qquad (A20)$$

For solving the perturbation equations it is required to know how the fluctuation in pressure δp is related to the fluctuation in the energy density $\delta\rho$. This is in general determined by the Lagrangian of the matter field.

If the perturbations are such that uniform density gauge coincide with the uniform pressure gauge, then such perturbations are known as adiabatic perturbations. This means that for adiabatic perturbations, we can choose ξ^o in Eqs.(A12) and (A13) such that in the new gauge $\widetilde{\delta\rho} = \widetilde{\delta p} = 0$. This implies that for adiabatic perturbations in any arbitrary gauge

$$\delta p = c_a^2\delta\rho \qquad (A21)$$

where c_a^2 is the adiabatic sound speed given by:

$$c_a^2 = \frac{\dot{\bar{p}}}{\dot{\bar{\rho}}} \qquad (A22)$$

This relation is true when the universe is dominated by a single perfect fluid but does not in general holds for scalar fields.

In general, the pressure fluctuation can be described as

$$\delta p = c_a^2\delta\rho + \delta p_{nad} \qquad (A23)$$

where δp_{nad} is the non adiabatic pressure fluctuation which is gauge invariant according to gauge transformations (A12) and (A13).

1. Effective speed of sound

In non relativistic fluid mechanics, the speed of sound is given by $c_s^2 = \delta p/\delta\rho$. The sound speed is thus determined by the property of the fluid $p = f(\rho)$.

However, in cosmological perturbation theory, the gauge transformations given by Eqs.(A12) and (A13) imply that the ratio $\delta p/\delta\rho$ would be gauge dependent. The speed of sound must be gauge invariant so that it is solely determined by the property of the fluid not dependent on the choice of the gauge. We expect that the speed of sound be the ratio of some form of the gauge invariant pressure fluctuation to the gauge invariant density fluctuation.

It is not possible to construct gauge invariant $\delta\rho$ and δp solely from variables describing matter perturbations $\delta\rho$, δp and u using Eqs.(A12) to (A14). However, if we include gauge transformation for metric perturbation B using Eq.(A10), then we can construct the following gauge invariant density fluctuation $\delta\rho$ and pressure fluctuation δp :

$$^{(gi)}\delta\rho = \delta\rho + a\dot{\rho}\left[au + B\right] \qquad (A24)$$

$$^{(gi)}\delta p = \delta p + a\dot{p}\left[au + B\right] \qquad (A25)$$

Using this we define the gauge invariant effective sound speed c_e^2 as :

$$c_e^2 = \frac{^{(gi)}\delta p}{^{(gi)}\delta\rho} = \frac{\delta p + a\dot{p}\left[au + B\right]}{\delta\rho + a\dot{\rho}\left[au + B\right]} \qquad (A26)$$

Gauge transformations (A10) and (A14) allows us to define a gauge in which $B = u = 0$. This gauge is known as the rest frame gauge because in this gauge the peculiar velocity $\delta u^i = 0$ and the perturbed energy momentum tensor becomes diagonal $i.e$ $\delta T^\mu{}_\nu = 0$ for all $\mu \neq \nu$. With the density and the pressure fluctuation in the rest gauge given by $\delta\rho_{rest}$ and δp_{rest} respectively, Eq.(A26) implies that $c_e^2 = \delta p_{rest}/\delta\rho_{rest}$. Hence the gauge invariant effective sound speed c_e^2 can be physically interpreted as the sound speed in the rest frame gauge in which the peculiar velocity δu^i is zero.

For a perfect fluid with adiabatic perturbations, which means that $\delta p = c_a^2\delta\rho$, Eq.(A26) implies that $c_e^2 = c_a^2$. Hence for a perfect fluid, there is only one sound speed and that is c_a^2 given by Eq.(A22). However, this is not true for scalar fields.

2. A general non canonical scalar field

A general non canonical scalar field ϕ has a Lagrangian of the form :

$$\mathcal{L} = \mathcal{L}(X, \phi) \tag{A27}$$

where

$$X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \tag{A28}$$

For this Lagrangian, the energy momentum tensor is given by :

$$T^\mu{}_\nu = \frac{\partial\mathcal{L}}{\partial X}\partial^\mu\phi\partial_\nu\phi - \mathcal{L}\delta^\mu{}_\nu \tag{A29}$$

Perturbation in the scalar field is defined as

$$\phi(\vec{x}, t) = \bar{\phi}(t) + \delta\phi(\vec{x}, t) \tag{A30}$$

For the background space time we can associate the following density $\bar{\rho}$ and pressure $\bar{p}$:

$$\bar{\rho} = 2\frac{\partial\bar{\mathcal{L}}}{\partial\bar{X}}\bar{X} - \bar{\mathcal{L}}(\bar{X}, \bar{\phi}) \tag{A31}$$

$$\bar{p} = \bar{\mathcal{L}}(\bar{X}, \bar{\phi}) \tag{A32}$$

where $\bar{X} = \frac{1}{2}\dot{\bar{\phi}}^2$ and $\bar{\mathcal{L}} = \bar{\mathcal{L}}(\bar{X}, \bar{\phi})$ is the Lagrangian of the background field obtained by treating the scalar field as a function only of time in Eq.(A27). For example, for canonical scalar field $\bar{\mathcal{L}}(\bar{X}, \bar{\phi}) = \frac{1}{2}\dot{\bar{\phi}}^2 - V(\bar{\phi})$.

Considering the perturbations in the scalar fields defined in Eq.(A30), we can associate the following $\delta\rho$, δp and u for the scalar field with the Lagrangian of the form Eq.(A27) :

$$\delta\rho = \left(\dot{\bar{\phi}}\dot{\delta\phi} - A\dot{\bar{\phi}}^2\right)\left[\frac{\partial\bar{\mathcal{L}}}{\partial\bar{X}} + 2\bar{X}\frac{\partial^2\bar{\mathcal{L}}}{\partial\bar{X}^2}\right]$$
$$- \left[\frac{\partial\bar{\mathcal{L}}}{\partial\bar{\phi}} - 2\bar{X}\frac{\partial^2\bar{\mathcal{L}}}{\partial\bar{X}\partial\bar{\phi}}\right]\delta\phi \tag{A33}$$

$$\delta p = \left(\dot{\bar{\phi}}\dot{\delta\phi} - A\dot{\bar{\phi}}^2\right)\frac{\partial\bar{\mathcal{L}}}{\partial\bar{X}} + \frac{\partial\bar{\mathcal{L}}}{\partial\bar{\phi}}\delta\phi \tag{A34}$$

$$u = -\frac{\delta\phi}{a^2\dot{\bar{\phi}}} - \frac{B}{a} \tag{A35}$$

The above equation [Eq.(A35)] implies that u for a Lagrangian of the form Eq.(A27) is the same as that for a canonical scalar field. Substituting Eqs.(A33) to (A35) in Eq.(A26), we find that for a Lagrangian of the form of Eq.(A27), effective speed of sound is given by [22] :

$$c_e^2 = \frac{\frac{\partial\bar{\mathcal{L}}}{\partial\bar{X}}}{\frac{\partial\bar{\mathcal{L}}}{\partial\bar{X}} + 2\bar{X}\frac{\partial^2\bar{\mathcal{L}}}{\partial\bar{X}^2}} \tag{A36}$$

Equation (A36) implies that for canonical scalar fields $c_e^2 = 1$[37].

3. Equations of Perturbation

One of the gauge in which cosmological perturbations can be studied is the longitudinal gauge defined by $B = E = 0$. In this gauge, for both scalar field and for perfect fluid, Eq.(A18) implies that :

$$A_l = \psi_l \equiv \Phi \tag{A37}$$

where we have denoted the metric perturbation in the longitudinal gauge by Φ. Using Eq.(A26), we find that in this gauge, the pressure fluctuation δp is related to the density fluctuation $\delta\rho$ as :

$$\delta p = c_e^2\delta\rho - 3H(\bar{\rho} + \bar{p})a^2u\left[c_e^2 - c_a^2\right] \tag{A38}$$

This is a general relation between the pressure fluctuation and the density fluctuation. For perfect fluid with constant equation of state parameter $c_e^2 = c_a^2 = w$, which from Eq.(A38) implies that $\delta p = w\delta\rho$. However, for scalar fields, in general $c_e^2 \neq c_a^2$.

The relation Eq.(A38) closes the equation of perturbations Eqs.(A15), (A19) and (A20). Defining the fractional density perturbations as $\delta \equiv \delta\rho/\bar{\rho}$, Eqs.(A19) and (A20), together with Eq.(A15) in longitudinal gauge would become :

$$\dot{\Phi} = -H\Phi - \frac{k^2\Phi}{3Ha^2} - \frac{4\pi G}{3H}\bar{\rho}\delta \tag{A39}$$

$$\dot{\delta} = (1 + w)k^2u + 3H\left(w - c_e^2\right)\delta + 9H^2$$
$$\times (1 + w)\left[c_e^2 - c_a^2\right]a^2u + 3(1 + w)\dot{\Phi} \tag{A40}$$

$$\dot{u} = -H\left(2 - 3c_e^2\right)u - \frac{c_e^2\delta}{a^2(1 + w)} - \frac{\Phi}{a^2} \tag{A41}$$

These three equations form a close set of equations if the universe is dominated by a single perfect field or a single scalar field. In case of perfect fluid $c_e^2 = c_a^2$. However, for scalar fields c_e^2 is given by Eq.(A36).

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